IDENTIFICATION FOR GENERAL DEGENERATE PROBLEMS OF HYPERBOLIC TYPE

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Abstract. A degenerate identification problem in Hilbert space is described, improving a previous paper [2]. An application to second order evolution equations of hyperbolic type is given. The abstract results are applied to concrete differential problems of interest in applied sciences.

Sunto. Un problema degenere di identificazione in uno spazio d Hilbert viene descritto, generalizzando un precedente lavoro di Favini e Marinoschi, [2]. Viene descritta una applicazione ad equazioni di evoluzione di secondo ordine di tipo iperbolico. I risultati astratti sono applicati ai problemi differenziali concreti di interesse in fisica matematica e scienze applicate.

1. Introduction

We recall that Favini and Marinoschi (see [2]) treated the inverse problem in a Hilbert space Y, of identifying $(y,f) \in C([0,\tau];D(L)) \times C([0,\tau];\mathbb{C})$ such that

$$M^* \frac{d}{dt}(My) = Ly + f(t)M^*z, \ 0 \le t \le \tau, \tag{1}$$

$$(My)(0) = My_0, \tag{2}$$

$$\Phi[My(t)] = g(t), \ 0 \le t \le \tau, \tag{3}$$

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where $M \in \mathcal{L}(Y)$ and L is a closed linear operator in Y satisfying

$$\Re(Lu,u)_Y \leq \beta\|Mu\|^2,\ \beta \in \mathbb{R},\ y \in D(L),$$
$$\text{range}(\lambda_0 M^*M - L) = Y,$$
$$\text{and } (\lambda_0 M^*M - L)^{-1} \in \mathcal{L}(Y),\ \lambda_0 > \beta,$$

z is a fixed element in Y, $y_0 \in D(L)$, $g \in C([0,\tau];\mathbb{C})$. Such research was motivated by a related existence and uniqueness of solution to (1), (2) in Favini–Yagi monograph [4]. In this paper, we want to handle the much more complicated equation

$$M^* \frac{d}{dt}(My) = Ly + f(t)z,\ 0 \leq t \leq \tau, \tag{4}$$

together with (2) and (3). Favini and Yagi, in their monograph, supposed that if a change of argument $y = x - f(t)L^{-1}z$ is done, then thanks to the invertibility of L, system (4)-(2), (3) transforms into

$$M^* \frac{d}{dt}(Mx) = Lx + f'(t)M^*ML^{-1}z,\ 0 \leq t \leq \tau, \tag{5}$$

$$(Mx)(0) = My_0 + f(0)ML^{-1}z, \tag{6}$$

$$\Phi[(Mx)(t)] = g(t) + \Phi[ML^{-1}z]f(t),\ 0 \leq t \leq \tau. \tag{7}$$

Main results are to be found in reference [3].

Although (5) reduces to the form (1), the unknown $f(t)$ appears in (6)-(7) and so a new method must be applied.

2. Main results

As a first step, we observe that (5) translates into the inclusion

$$\frac{d}{dt}(Mx) - f'(t)ML^{-1}z \in M^{*-1}Lx. \tag{8}$$

Changing the variable to $x = L^{-1}M^*\xi$ in (8) we obtain the equivalent problem, where $T = ML^{-1}M^*$, that is

$$\frac{d}{dt}T\xi = \xi + f'(t)ML^{-1}z, \ 0 \le t \le \tau, \tag{9}$$

$$T\xi(0) = My_0 + f(0)ML^{-1}z, \tag{10}$$

$$\Phi[T\xi(t)] = g(t) + f(t)\Phi[ML^{-1}z]. \tag{11}$$

Notice that we know (see Favini and Marinoschi [2]) that $\widetilde{T}$ the restriction $\widetilde{T}$ of T to $\overline{R(T)}$ is invertible

$$\widetilde{T}^{-1} : \overline{R(T)} \to \overline{R(T)}$$

and generates a C_0 semigroup in $\overline{R(T)}$. Moreover, $Y = N(T) \oplus \overline{R(T)}$. Denote by P the projection operator on $N(T)$ along $\overline{R(T)}$. Equality (10) implies that My_0 must belong to $R(T) \subseteq \overline{R(T)}$, $ML^{-1}z \in \overline{R(T)}$, so that (10) reads

$$T\xi(0) = \widetilde{T}(1-P)\xi(0) = (1-P)My_0 + f(0)(1-P)ML^{-1}z. \tag{12}$$

This implies that (9)-(11) reads

$$\frac{d}{dt}\widetilde{T}(1-P)\xi = (1-P)\xi + f'(t)(1-P)ML^{-1}z, \ 0 \le t \le \tau, \tag{13}$$

$$\widetilde{T}(1-P)\xi(0) = (1-P)My_0 + f(0)(1-P)ML^{-1}z, \tag{14}$$

$$\Phi[\widetilde{T}(1-P)\xi(t)] = g(t) + \Phi[(1-P)ML^{-1}z], \tag{15}$$

together with

$$0 = P\xi(t) + f(t)PML^{-1}z.$$

But $PML^{-1}z = 0$ provided that $ML^{-1}z \in \overline{R(T)}$, so that

$$P\xi(t) = 0. \tag{16}$$

In view of relations (13)-(15), we have necessarily

$$\Phi[\xi(t)] = \Phi[(1-P)\xi(t)] = g'(t).$$

We see that appropriate regularity of the data is necessary. Denote

$$\widetilde{T}(1-P)\xi = \eta.$$

Then problem (13)-(15) becomes

$$\frac{d}{dt}\eta = \widetilde{T}^{-1}\eta + f'(t)(1-P)ML^{-1}z, \ 0 \le t \le \tau, \tag{17}$$

$$\eta(0) = (1-P)My_0 + f(0)(1-P)ML^{-1}z, \tag{18}$$

$$\Phi[\eta(t)] = g(t) + \Phi[(1-P)ML^{-1}z]f(t). \tag{19}$$

We specify that f must be more regular in order to solve (17), without the assumption $(1-P)ML^{-1}z \in D(\widetilde{T}^{-1}) = R(T)$. Moreover, the compatibility relation $\Phi[My_0] = g(0)$ must hold (see (18), (19)). The main result in the paper is the following.

Theorem 1. *Suppose that*

$$ML^{-1}z \in \overline{R(T)},$$

$$\Phi[My_0] = \Phi[(1-P)My_0] = g(0),$$

$$\Phi[\widetilde{T}^{-1}(1-P)ML^{-1}z] \neq 0,$$

$$g \in C^3([0,\tau],\mathbb{C}),$$

$$\sup_{t>0} \|\widetilde{T}^{-2} e^{t\widetilde{T}^{-1}} \widetilde{T}^{-1}(1-P)ML^{-1}z\| < \infty, \tag{20}$$

$$\sup_{t>0} \|\widetilde{T}^{-2} e^{t\widetilde{T}^{-1}} \widetilde{T}^{-1}(1-P)My_0\| < \infty. \tag{21}$$

hold. Thus, the inverse problem (5)-(7) *admits a unique solution*

$$(y,f) \in C([0,\tau];D(L)) \times C^2([0,\tau];\mathbb{C}).$$

Proof. Under more regularity assumptions, to be made precise below,

$$\begin{aligned} \eta(t)(=\widetilde{T}(1-P)\xi(t)) &= e^{t\widetilde{T}^{-1}}(1-P)My_0 + e^{t\widetilde{T}^{-1}} f(0)(1-P)ML^{-1}z \\ &\quad + \int_0^t e^{(t-s)\widetilde{T}^{-1}}(1-P)ML^{-1}zf'(s)ds. \end{aligned} \tag{22}$$

Integrating by parts,

$$\begin{aligned} \eta(t) &= e^{t\widetilde{T}^{-1}}(1-P)My_0 + e^{t\widetilde{T}^{-1}} f(0)(1-P)ML^{-1}z \\ &\quad + f(t)(1-P)ML^{-1}z - e^{t\widetilde{T}^{-1}}(1-P)ML^{-1}zf(0) \\ &\quad + \int_0^t e^{(t-s)\widetilde{T}^{-1}} \widetilde{T}^{-1}(1-P)ML^{-1}zf(s)ds \\ &= e^{t\widetilde{T}^{-1}}(1-P)My_0 + f(t)(1-P)ML^{-1}z \\ &\quad + \int_0^t e^{(t-s)\widetilde{T}^{-1}} \widetilde{T}^{-1}(1-P)ML^{-1}zf(s)ds. \end{aligned} \tag{23}$$

Therefore, relation (19) implies that necessarily

$$\begin{aligned}&\Phi[e^{t\widetilde{T}^{-1}}(1-P)My_0]+f(t)\Phi[(1-P)ML^{-1}z]\\&\quad+\int_0^t\Phi[e^{(t-s)\widetilde{T}^{-1}}\widetilde{T}^{-1}(1-P)ML^{-1}z]f(s)ds\\&\quad=g(t)+f(t)\Phi[(1-P)ML^{-1}z],\end{aligned}$$

that is, the integral equation of the second kind

$$\int_0^t\Phi[e^{(t-s)\widetilde{T}^{-1}}\widetilde{T}^{-1}(1-P)ML^{-1}z]f(s)ds=g(t)-\Phi[e^{t\widetilde{T}^{-1}}(1-P)My_0] \tag{24}$$

must hold. Notice that $g(0)-\Phi[(1-P)My_0]=0$ in view of our assumptions. Deriving both members in (24), we get

$$\begin{aligned}&f(t)\Phi[\widetilde{T}^{-1}(1-P)ML^{-1}z]\\&\quad+\int_0^t\Phi[\widetilde{T}^{-1}e^{(t-s)\widetilde{T}^{-1}}\widetilde{T}^{-1}(1-P)ML^{-1}z]f(s)ds\\&\quad=g'(t)-\Phi[\widetilde{T}^{-1}e^{t\widetilde{T}^{-1}}(1-P)My_0].\end{aligned} \tag{25}$$

If $\Phi[\widetilde{T}^{-1}(1-P)ML^{-1}z]\neq 0$, such an equation admits a unique solution (also global, see Lorenzi's monograph [5]), $f\in C([0,\tau];\mathbb{C})$. Of course, condition that

$$\sup_{t>0}\|\widetilde{T}^{-1}e^{t\widetilde{T}^{-1}}\widetilde{T}^{-1}(1-P)ML^{-1}z\|<\infty$$

must hold, i.e., if $\widetilde{T}^{-1}$ generates an analytic semigroup then $\widetilde{T}^{-1}(1-P)ML^{-1}z$ belongs to the Favard space F_1 for the operator $\widetilde{T}^{-1}$, (according to the Engel-Nagel monograph [1]). We have, by differentiating (25):

$$\begin{aligned}&f'(t)\Phi[\widetilde{T}^{-1}(1-P)ML^{-1}z]+\frac{d}{dt}\int_0^t\Phi[\widetilde{T}^{-1}e^{s\widetilde{T}^{-1}}\widetilde{T}^{-1}(1-P)ML^{-1}z]f(t-s)ds\\&\quad=g''(t)-\Phi[\widetilde{T}^{-1}e^{t\widetilde{T}^{-1}}\widetilde{T}^{-1}(1-P)My_0],\end{aligned}$$

whence

$$
\begin{aligned}
&f'(t)\Phi[\widetilde{T}^{-1}(1-P)ML^{-1}z] + \Phi[\widetilde{T}^{-1}e^{t\widetilde{T}^{-1}}\widetilde{T}^{-1}(1-P)ML^{-1}z]f(0)\\
&\quad + \int_0^t \Phi[\widetilde{T}^{-1}e^{s\widetilde{T}^{-1}}\widetilde{T}^{-1}(1-P)ML^{-1}z]f'(t-s)ds\\
&\quad = g''(t) - \Phi[\widetilde{T}^{-1}e^{t\widetilde{T}^{-1}}\widetilde{T}^{-1}(1-P)My_0].
\end{aligned}
\tag{26}
$$

Notice that in a first step, one observes that

$$
\begin{aligned}
&f'(t)\Phi[\widetilde{T}^{-1}(1-P)ML^{-1}z] + \Phi[\widetilde{T}^{-1}e^{t\widetilde{T}^{-1}}\widetilde{T}^{-1}(1-P)ML^{-1}z]f(0)\\
&\quad + \int_0^t \Phi[\widetilde{T}^{-1}e^{(t-s)\widetilde{T}^{-1}}\widetilde{T}^{-1}(1-P)ML^{-1}z]f'(s)ds\\
&\quad = g''(t) - \Phi[\widetilde{T}^{-1}e^{t\widetilde{T}^{-1}}\widetilde{T}^{-1}(1-P)My_0],
\end{aligned}
\tag{27}
$$

so that we get an integral equation solvable in $f'(t)$ provided that

$$
\sup_{t>0} \|\widetilde{T}^{-1}e^{t\widetilde{T}^{-1}}\widetilde{T}^{-1}(1-P)ML^{-1}z] < \infty
$$

$$
\sup_{t>0} \|\widetilde{T}^{-1}e^{t\widetilde{T}^{-1}}\widetilde{T}^{-1}(1-P)My_0\| < \infty.
$$

Thus, f is differentiable and $f'(t)$ satisfies (27). Having this problem, one reaches the desired regularity for the unknown f. On the other hand, we can derive one time more in (27), obtaining

$$
\begin{aligned}
f''(t)\Phi[\widetilde{T}^{-1}(1-P)ML^{-1}z] &= -\Phi[\widetilde{T}^{-2}e^{t\widetilde{T}^{-1}}\widetilde{T}^{-1}(1-P)ML^{-1}z]f(0)\\
&\quad +\Phi[\widetilde{T}^{-2}(1-P)ML^{-1}z]f'(t)\\
&\quad + \int_0^t \Phi[\widetilde{T}^{-2}e^{(t-s)\widetilde{T}^{-1}}\widetilde{T}^{-1}(1-P)ML^{-1}z]f'(s)ds\\
&\quad +g'''(t) - \Phi[\widetilde{T}^{-2}e^{t\widetilde{T}^{-1}}\widetilde{T}^{-1}(1-P)My_0].
\end{aligned}
\tag{28}
$$

Of course, here we use (20) and (21). By (22) and (16) we have

$$\begin{aligned}(1-P)\xi(t) &= \widetilde{T}^{-1}e^{t\widetilde{T}^{-1}}(1-P)My_0 + f(t)\widetilde{T}^{-1}(1-P)ML^{-1}z \\ &\quad + \int_0^t \widetilde{T}^{-1}e^{(t-s)\widetilde{T}^{-1}}\widetilde{T}^{-1}(1-P)ML^{-1}zf(s)ds, \\ P\xi(t) &= 0,\end{aligned}$$

so that

$$\begin{aligned}g'(t) &= \Phi[\widetilde{T}^{-1}e^{t\widetilde{T}^{-1}}(1-P)My_0] \\ &\quad + f(t)\Phi[\widetilde{T}^{-1}(1-P)ML^{-1}z] \\ &\quad + \int_0^t \Phi[\widetilde{T}^{-1}e^{(t-s)\widetilde{T}^{-1}}\widetilde{T}^{-1}(1-P)ML^{-1}z]f(s)ds \\ &= \Phi[(1-P)\xi(t)] = \Phi[\xi(t)],\end{aligned}$$

as desired. □

Example 1. As a trivial but enlightening example, take

$$M = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad L = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

so that

$$M^* = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

and thus

$$MM^* = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = A,$$

$$N(A) = \{(0,0,w),\, w \in \mathbb{C}\},\ R(A) = \overline{R(A)} = \{(y,x,0),\, y,x \in \mathbb{C}\},$$

$$P = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad 1 - P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Therefore, $(1-P)M = M$, as declared. A direct calculation allows to solve directly the identification problem

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} y \\ x \\ w \end{bmatrix} = \begin{bmatrix} y \\ x \\ w \end{bmatrix} + f(t) \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix},$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} y \\ x \\ w \end{bmatrix} (0) = \begin{bmatrix} y_0 + w_0 \\ z_0 \\ 0 \end{bmatrix},$$

$$\Phi \left(\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} y \\ x \\ w \end{bmatrix} (t) \right) = g(t),$$

where $\Phi = [1\ 1\ 0]$. We get

$$\frac{d}{dt} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} y \\ x \\ w \end{bmatrix} = \begin{bmatrix} y \\ x \\ w \end{bmatrix} + f(t) \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix},$$

$$y + x + w = g(t),$$

i.e.,

$$\begin{aligned} 2(y+w)' &= y+w+f(t)(z_1+z_3) \\ x' &= x+f(t)z_2, \end{aligned}$$

which implies

$$\begin{aligned} x &= e^t x_0 + \int_0^t e^{t-s} f(s) z_2 ds, \\ y+w &= e^{t/2}(y_0+z_0) + \int_0^t \frac{1}{2} e^{(t-s)/2} f(s)(z_1+z_3) ds, \end{aligned}$$

so that all is reduced to solve

$$\int_0^t e^{t-s} f(s) z_2 ds + \frac{1}{2}\int_0^t e^{(t-s)/2} f(s)(z_1+z_3) ds + e^t x_0 + e^{t/2}(y_0+z_0) = g(t).$$

Therefore, the integral equation

$$\begin{aligned} &\left(\frac{z_1+z_3}{2}+z_2\right) f(t) + \int_0^t e^{t-s} f(s) ds \\ &+\frac{1}{4}\int_0^t e^{(t-s)/2} f(s)(z_1+z_3) ds \\ &+e^t x_0 + \frac{1}{2} e^{t/2}(y_0+z_0) \\ &= g'(t) \end{aligned}$$

is obtained. So, if $z_1 + z_3 + 2z_2 \neq 0$, such an equation is uniquely solvable. Concerning Theorem 1, observe that

$$\widetilde{A}^{-1}M \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} z_1 + z_3 \\ z_2 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{z_1+z_3}{2} \\ z_2 \\ 0 \end{bmatrix}$$

so that

$$\Phi \left[\widetilde{A}^{-1}M \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} \right] = \frac{z_1 + z_3}{2} + z_2$$

and the condition $z_1 + 2z_2 + z_3 \neq 0$ is obtained, as previously.

3. Equations of the second order in time

Our concern is the identification problem in the Hilbert space H with inner product $(\cdot, \cdot)_H$

$$C^{1/2}\frac{d}{dt}(C^{1/2}u') + B\frac{du}{dt} + A_H u = f(t)z, \ 0 \leq t \leq \tau,$$

$$u(0) = u_0, \quad C^{1/2}\frac{du}{dt}(0) = C^{1/2}u_1,$$

$$\Phi[C^{1/2}u(t)] = g(t),$$

where C is a bounded self-adjoint operator in H, $C \geq 0$, B is a closed linear operator in H, A is a linear bounded operator from another Hilbert space V continuously and densely embedded H. By identifying H with its dual space, we have $V \subset H \subset V'$. The scalar product between V and V' is denoted by $\langle \cdot, \cdot \rangle$. We assume $g \in C^1([0,\tau];\mathbb{C})$ and the compatibility relations

$$\Phi[C^{1/2}u_0] = g(0), \quad \Phi[C^{1/2}u_1] = g'(0).$$

Moreover, let A satisfy

$$\begin{aligned} \langle Au, v \rangle &= \langle u, Av \rangle, \quad u, v \in V, \\ \langle Au, u \rangle &\geq \omega \|u\|_V^2, \quad u \in V,\ \omega > 0. \end{aligned}$$

We recall that if A_H denotes the self-adjoint operator in H with the domain

$$D(A_H) = \{v \in V,\ Au \in H),\ A_H u = A_H,\ u \in D(A_H),$$

and $A^{1/2}$ is the square root of A_H, then $D(A^{1/2}) = V$. One also assumes that

$$V = D(A^{1/2}) \subset D(B)$$

$$\mathrm{Re}\,\langle Bv, v \rangle \geq 0, \quad \text{for all } v \in V.$$

Then, the previous identification problem can be written in the form

$$\begin{bmatrix} 1 & 0 \\ 0 & C^{1/2} \end{bmatrix} \frac{d}{dt} \begin{bmatrix} 1 & 0 \\ 0 & C^{1/2} \end{bmatrix} \begin{bmatrix} u \\ u' \end{bmatrix} + \begin{bmatrix} 0 & -1 \\ A_H & B \end{bmatrix} \begin{bmatrix} u \\ u' \end{bmatrix}$$

$$= f(t) \begin{bmatrix} 0 \\ z \end{bmatrix}, \ 0 \leq t \leq \tau,$$

$$\begin{bmatrix} I & 0 \\ 0 & C^{1/2} \end{bmatrix} \begin{bmatrix} u(0) \\ u'(0) \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & C^{1/2} \end{bmatrix} \begin{bmatrix} u_0 \\ u_1 \end{bmatrix},$$

$$\widetilde{\Phi}\left(\begin{bmatrix} 1 & 0 \\ 0 & C^{1/2} \end{bmatrix}\begin{bmatrix} u(t) \\ u'(t) \end{bmatrix}\right) := \Phi[C^{1/2}\ u'(t)] = g(t),\ 0 \le t \le \tau.$$

in the ambient space $X = D(A^{1/2}) \times H$, with the inner product

$$\langle (x, y), (x_1, y_1)\rangle_X = (A^{1/2}x, A^{1/2}x_1)_H + (y, y_1)_H,\ (x, y), (x_1, y_1) \in X.$$

It is seen that the operators M and L from X into itself defined by

$$D(M) = X,\ M(x, y) = (x, C^{1/2}y),\ (x, y) \in X,$$

$$D(L) = D(A_H) \times D(A^{1/2}),\ L(x, y) = (y, -A_H x - By),\ (x, y) \in D(L),$$

satisfy all assumptions in Section 2, with $\beta = 0$. The details are left to the reader.

4. Examples and applications

Example 2. Consider the identification problem

$$\frac{\partial m(x)\nu}{\partial t} = -\frac{\partial \nu}{\partial x} + f(t)z(t), \quad \infty < x < \infty,\ t \ge 0,$$

$$m(x)\nu(x, 0) = m(t)\nu_0(x)$$

$$\int_{-\infty}^{\infty} \eta(x)m(x)\nu(x, t)dx = g(t), \quad 0 \le t \le \tau.$$

Here $m(x)$ is the characteristic function of the set $J = (-0, a) \cup (b, x_0)$, so that

$$\frac{\partial m(x)\nu}{\partial t} = m(x)D_t[m(x)\nu].$$

It is seen in Favini-Marinoschi [2] that all conditions in theorem 1 are satisfied.

Example 3. The general Maxwell equations, with arbitrary term $f(t)z$ instead of $f(t)Mz$ can be considered generalizing Example 2 in Favini-Marinoschi [2].

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